Dimensionality Reduction for Spatial-Temporally Distributed Data

Presented by Yueyu Hu, Zibo Ye

Outline

- Background
- Selection: Action Synopsis
- Manipulation: Motion Synthesis and Editing
- Conclusion

Background

Why?

- To remove some parts of the data
  - Redundancy (We don’t need)
  - Irrelevance (We don’t care)
- To get an abstract of the data
  - Saliency analysis
  - Efficient Manipulation

Where?

- High-Dimension Spatial Representations
  - 3D meshes
  - Point Cloud
  - Images (Light Field)
- Temporal-Distributed Representations
  - Motion of Poses and Pixels
  - Animations

Selection: Action Synopsis

Assa J, Caspi Y, Cohen-Or D. Action synopsis: pose selection and illustration, ACM Transactions on Graphics (TOG). ACM, 2005, 24(3): 667-676.

Synopsis: Motion in a Still Image

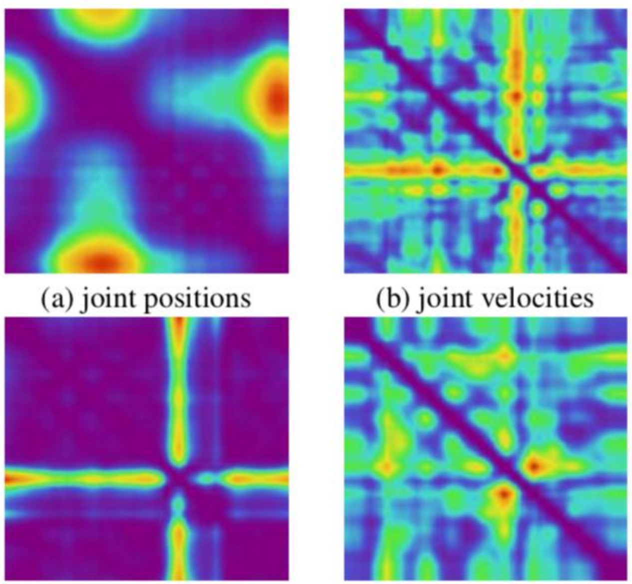
- Story Summary
- Action Recognition

Method

- Extracts Aspects
- Measuring Difference
  - Affinity matrix
- Dimensionality Reduction
  - Replicated Multi-Dimensional Scaling (RMDS)
- Pose Selection
  - Curve in the low dimensional space

Affinity Matrices of Aspects

- Position, Velocity, Angle & Angular Velocity
  - Affinity matrices to measure dynamics

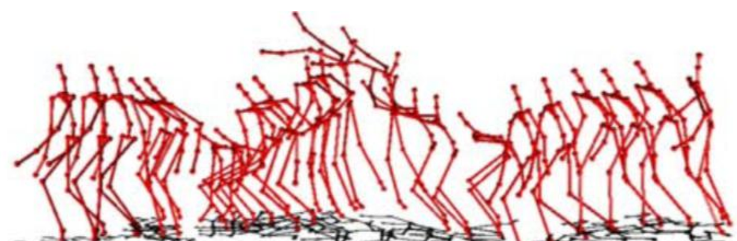


(a) joint positions (b) joint velocities

(c) joint angles (d) joint angular velocities

$$d_a(f_1, f_2) = \sum_{j \in joints} b_j \frac{(x_j^{f_1} - x_j^{f_2})^2}{\sigma_j^2},$$

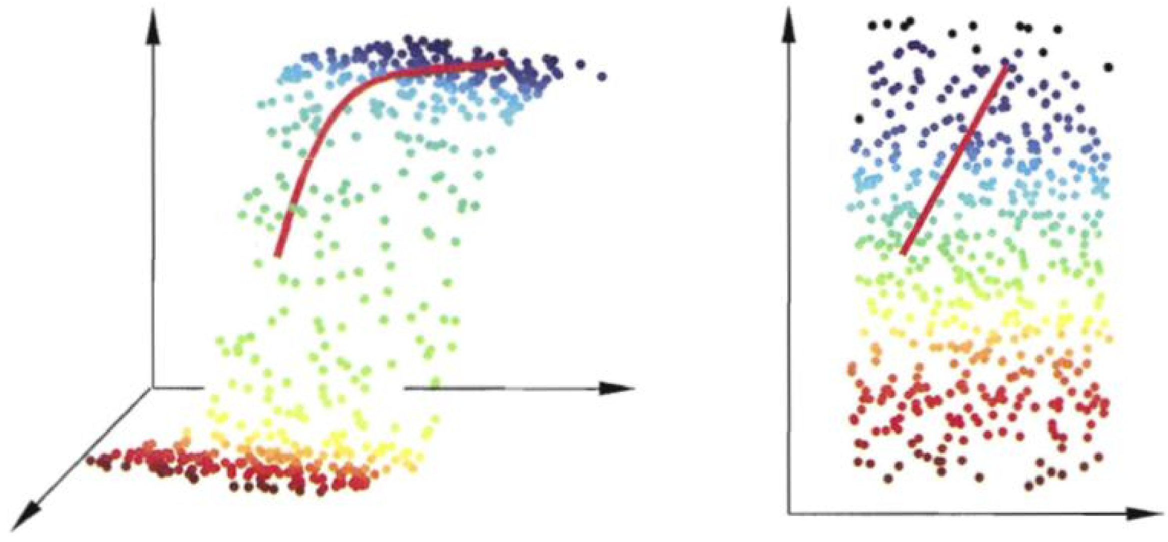
$$\hat{d}(f_1, f_2) = e^{-\frac{(|f_1 - f_2|)}{N}} d(f_1, f_2),$$



(a)

Multi-Dimensional Scaling (MDS)

- Given Distances of Samples
- Find Low Dimensional Representation
  - Preserve original distance



MDS

假定 m 个样本在原始空间的距离矩阵为 $\mathbf{D} \in \mathbb{R}^{m \times m}$, 其第 i 行 j 列的元素 $dist_{ij}$ 为样本 $\boldsymbol{x}_i$ 到 $\boldsymbol{x}_j$ 的距离. 我们的目标是获得样本在 d' 维空间的表示 $\mathbf{Z} \in \mathbb{R}^{d' \times m}$, $d' \leqslant d$, 且任意两个样本在 d' 维空间中的欧氏距离等于原始空间中的距离, 即 $\|\boldsymbol{z}_i - \boldsymbol{z}_j\| = dist_{ij}$.

令 $\mathbf{B} = \mathbf{Z}^{\mathrm{T}}\mathbf{Z} \in \mathbb{R}^{m \times m}$, 其中 $\mathbf{B}$ 为降维后样本的内积矩阵, $b_{ij} = \boldsymbol{z}_i^{\mathrm{T}}\boldsymbol{z}_j$, 有

$$\begin{aligned} dist_{ij}^2 &= \|\boldsymbol{z}_i\|^2 + \|\boldsymbol{z}_j\|^2 - 2\boldsymbol{z}_i^{\mathrm{T}}\boldsymbol{z}_j \\ &= b_{ii} + b_{jj} - 2b_{ij} \ . \end{aligned} \tag{10.3}$$

为便于讨论, 令降维后的样本 $\mathbf{Z}$ 被中心化, 即 $\sum_{i=1}^m \boldsymbol{z}_i = \mathbf{0}$. 显然, 矩阵 $\mathbf{B}$ 的行与列之和均为零, 即 $\sum_{i=1}^m b_{ij} = \sum_{j=1}^m b_{ij} = 0$. 易知

$$dist_{i\cdot}^2 = \frac{1}{m}\sum_{j=1}^m dist_{ij}^2 \ , \qquad dist_{\cdot j}^2 = \frac{1}{m}\sum_{i=1}^m dist_{ij}^2 \ , \qquad \sum_{i=1}^m\sum_{j=1}^m dist_{ij}^2 = 2m\,\mathrm{tr}(\mathbf{B}) \ ,$$

其中 $\mathrm{tr}(\cdot)$ 表示矩阵的迹(trace), $\mathrm{tr}(\mathbf{B}) = \sum_{i=1}^{m} \|\boldsymbol{z}_i\|^2$. 令

MDS

$$dist_{i\cdot}^2 = \frac{1}{m}\sum_{j=1}^{m} dist_{ij}^2 \ , \tag{10.7}$$

$$dist_{\cdot j}^2 = \frac{1}{m}\sum_{i=1}^{m} dist_{ij}^2 \ , \tag{10.8}$$

$$dist_{\cdot\cdot}^2 = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} dist_{ij}^2 \ , \tag{10.9}$$

由式(10.3)和式(10.4)~(10.9)可得

$$b_{ij} = -\frac{1}{2}(dist_{ij}^2 - dist_{i\cdot}^2 - dist_{\cdot j}^2 + dist_{\cdot\cdot}^2) \ , \tag{10.10}$$

由此即可通过降维前后保持不变的距离矩阵 $\mathbf{D}$ 求取内积矩阵 $\mathbf{B}$.

对矩阵 $\mathbf{B}$ 做特征值分解(eigenvalue decomposition), $\mathbf{B} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{\mathrm{T}}$, 其中 $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_d)$ 为特征值构成的对角矩阵, $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_d$, $\mathbf{V}$ 为特征向量矩阵. 假定其中有 d^* 个非零特征值, 它们构成对角矩阵 $\mathbf{\Lambda}_* = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{d^*})$, 令 $\mathbf{V}_*$ 表示相应的特征向量矩阵, 则 $\mathbf{Z}$ 可表达为

$$\mathbf{Z} = \mathbf{\Lambda}_*^{1/2}\mathbf{V}_*^{\mathrm{T}} \in \mathbb{R}^{d^* \times m} \ . \tag{10.11}$$

RMDS

- Metric MDS

$$\min_{\phi} \sum_{i,j} (e_{i,j} - d_{i,j})^2$$

- Non-Metric MDS
  - Keep Distance up to monotone increasing function

$$\min_{\phi} \sum_{i,j} w_{i,j} (f(e_{i,j}) - d_{ij})^2,$$

- Replicated MDS
  - Analyze multiple affinity matrices

$$\sum_{k} \sum_{\mathrm{i<j}} w_{ij}^{k} (f^{k}(e_{ij})) - d_{ij}^{k})^2.$$

Pose Selection

- Low-Dimensional Representation
  - Motion ⟶ Curve in low-dim space
- Smoothing / Averaging

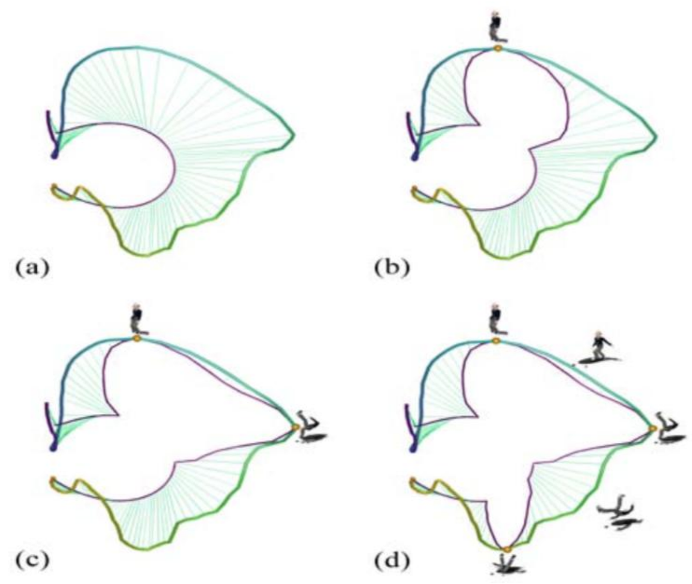
$$\bar{C}(p) = \sum_{i \in \delta} C(i) e^{-\frac{\|(p-i)\|^2}{\delta^2}} / \sum_{i \in \delta} e^{-\frac{\|(p-i)\|^2}{\delta^2}},$$

- Distance to Average

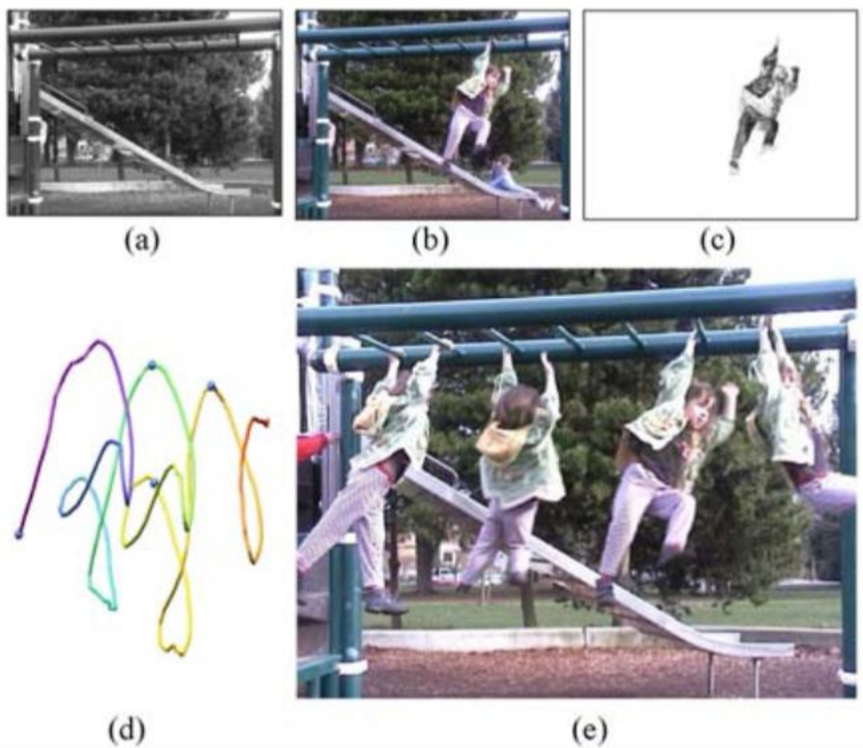
$$r_p = ||C(p) - \bar{C}(p)||.$$

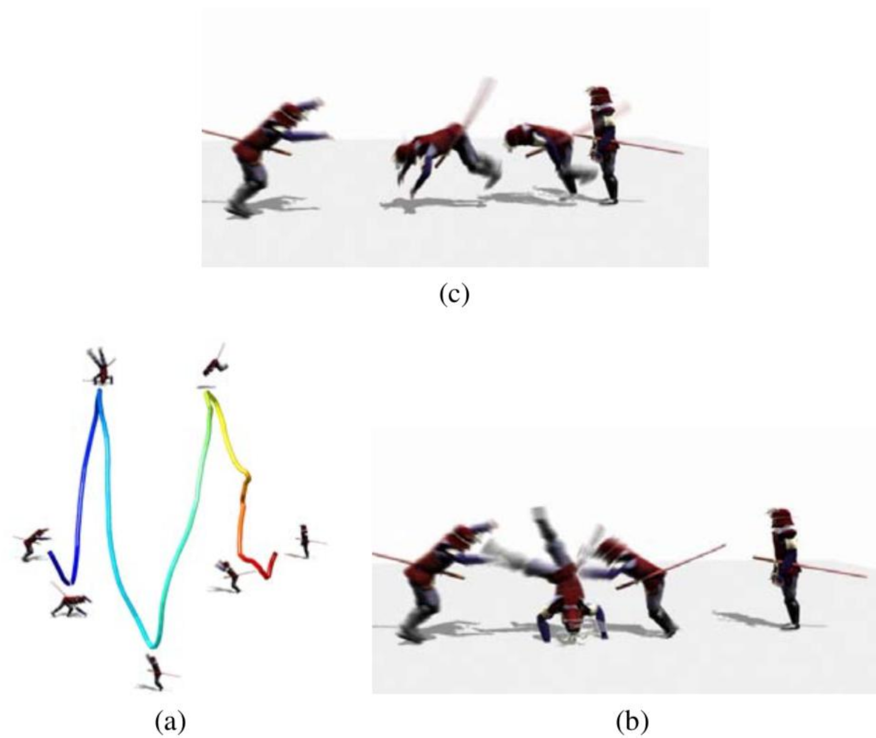
- Iterative Selection

$$\bar{C}(p) := \alpha C(p) + (1-\alpha)\bar{C}(p),$$



Experimental Results





Limitations

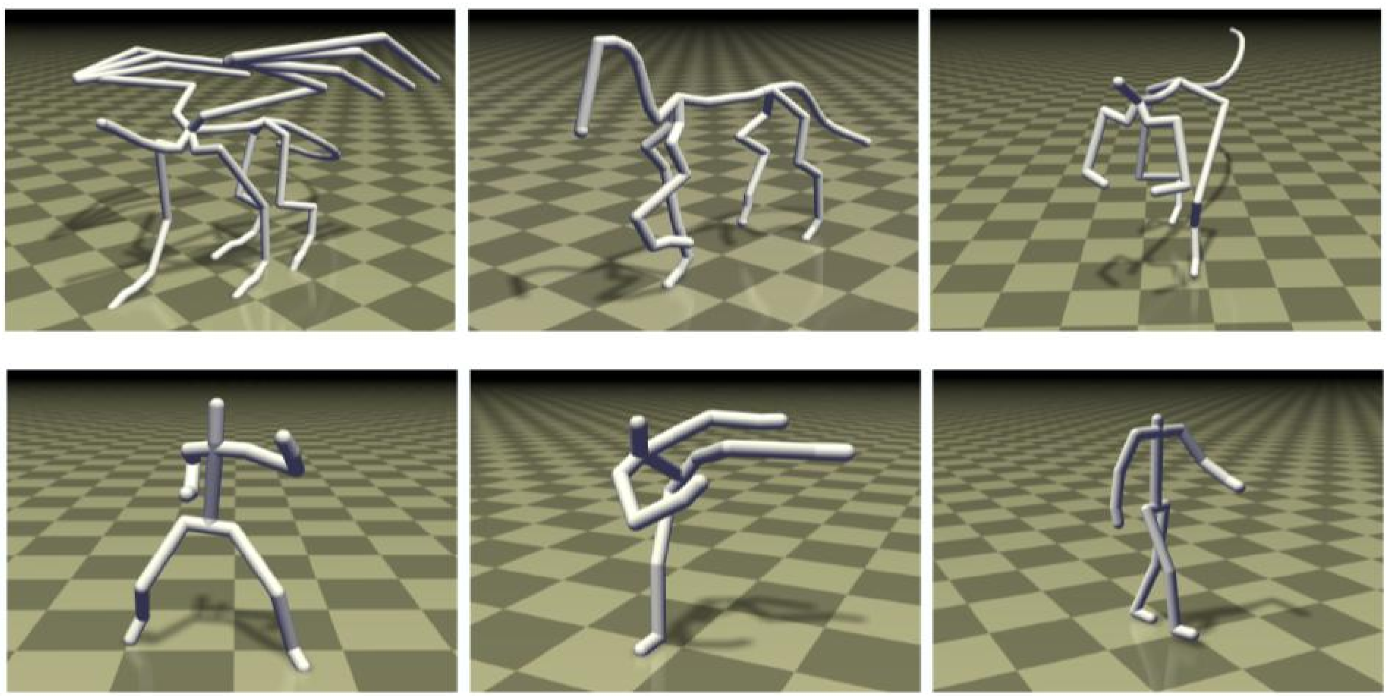
- Arbitrary Aspects
  - Neglecting co-occurrence features
- Only Bottom-Up Summarizing
  - No direct representation of motion
- General Method for Dimensionality Reduction

Manipulation: Motion Synthesis and Editing

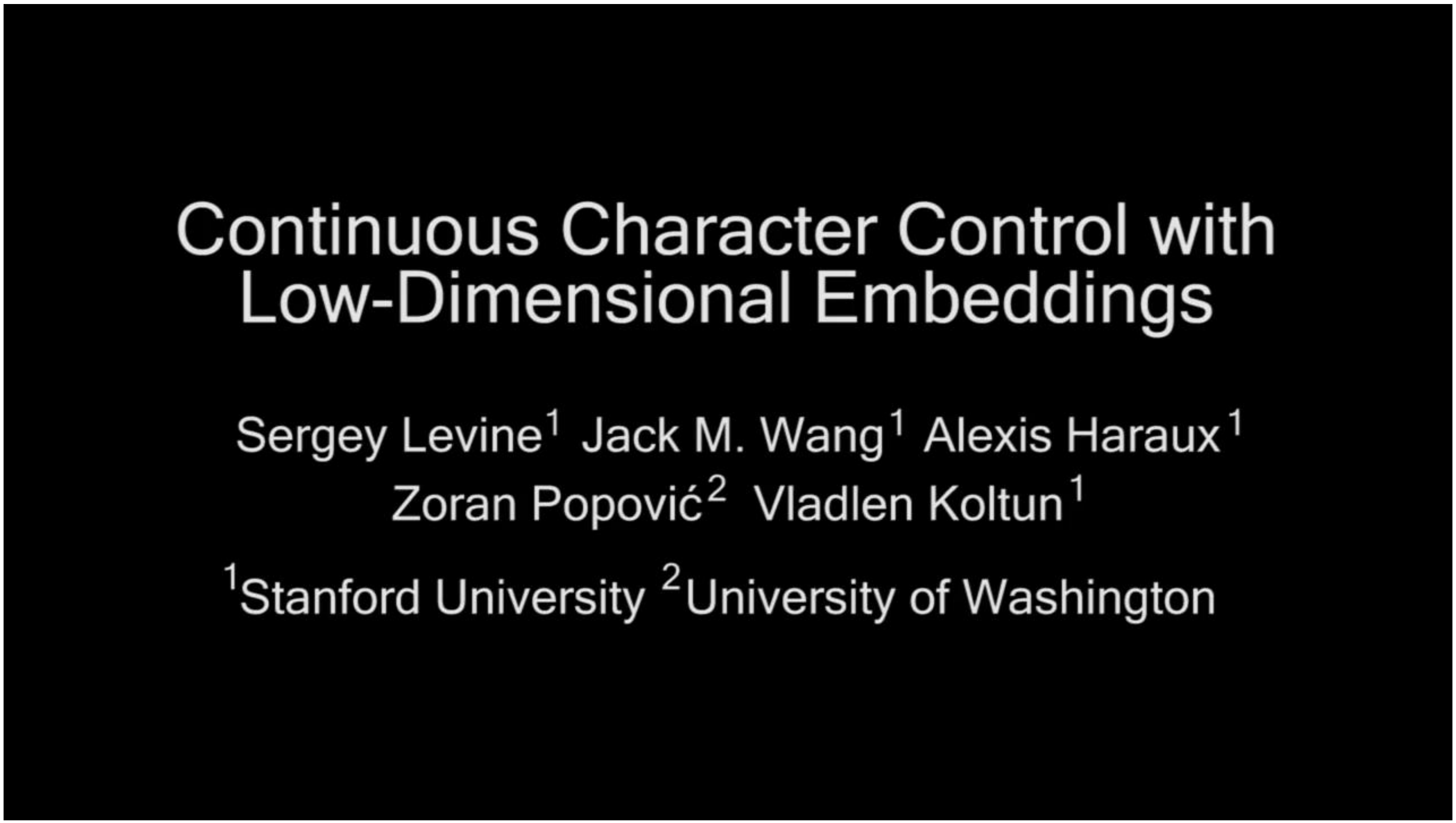
Levine S, Wang J M, Haraux A, et al. Continuous character control with low-dimensional embeddings. ACM Transactions on Graphics (TOG), 2012, 31(4): 28.

Continuous Pose Control

- Representations of Pose Sequences
- Manipulatable Embedding

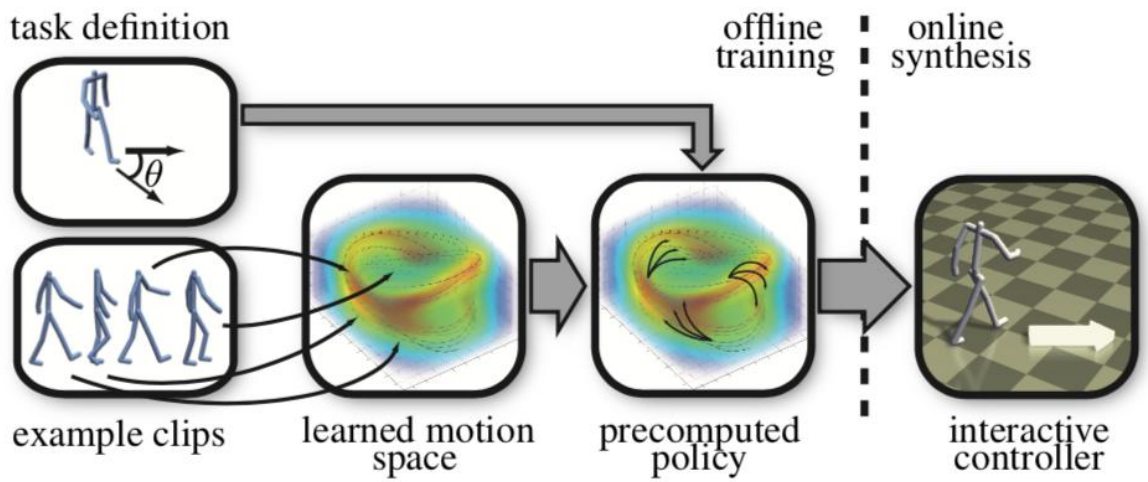


Visualization



Method

- Project Example Clips to Latent Space
  - Gaussian Process Latent Variable Model (GPLVM)
- Connectivity prior
- Pose Synthesis
- Controlling



GPLVM Preliminaries

- Distance Preserving Dim-Reduction (MDS)

$$S = \sum_{n=1}^{N} \sum_{m=n+1}^{N} w_{mn} \left(\delta_{mn} - d_{mn}\right)^2 ,$$

- Problems
  - Back-Projection to High Dimensional Space?
  - Missing Attributes?
  - Non-Linearity?
- PCA / Kernel PCA

GPLVM

- Problem: $\mathbf{y}_n = \mathbf{W}\mathbf{x}_n + \eta_n \qquad p(\eta_n) = N(\eta_n|\mathbf{0}, \beta^{-1}\mathbf{I}).$
- Idea of Probability PCA: $p(\mathbf{x}_n) = N(\mathbf{x}_n|\mathbf{0}, \mathbf{I}).$

$$p(\mathbf{y}_n|\mathbf{W}, \beta) = \int p(\mathbf{y}_n|\mathbf{x}_n, \mathbf{W}, \beta)\, p(\mathbf{x}_n)\, d\mathbf{x}_n = N(\mathbf{y}_n|\mathbf{0}, \mathbf{W}\mathbf{W}^{\mathrm{T}} + \beta^{-1}\mathbf{I}).$$

- Why $\mathbf{W}$?

$$p(\mathbf{y}_{:,d}|\mathbf{X}, \beta) = N(\mathbf{y}_{:,d}|\mathbf{0}, \mathbf{X}\mathbf{X}^{\mathrm{T}} + \beta^{-1}\mathbf{I}).$$

- Non-Linear Modification

$$k(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i^{\mathrm{T}}\mathbf{x}_j + \beta^{-1}\delta_{ij},$$

↘

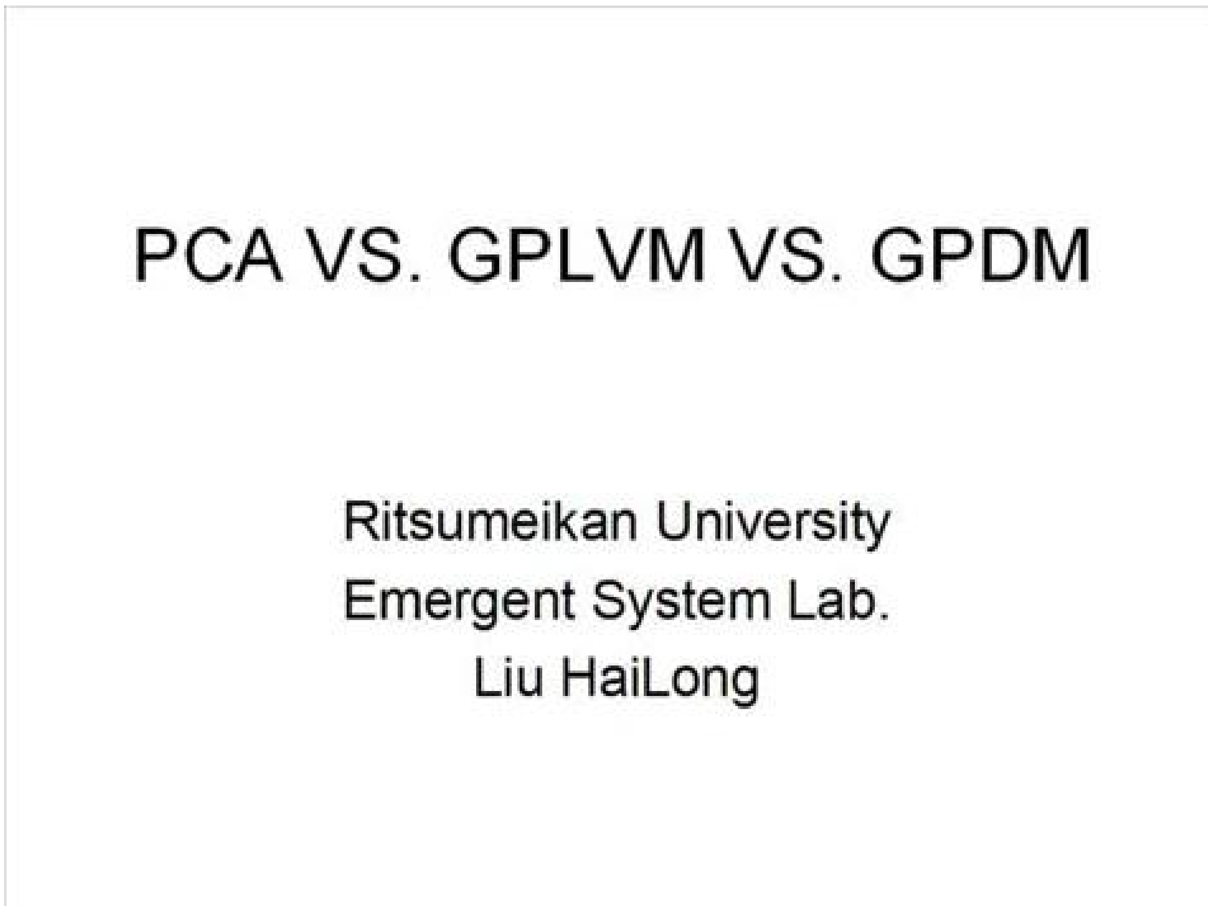
$$k_{\mathrm{rbf}}(\mathbf{x}_i, \mathbf{x}_j; \bar{\alpha}) = \alpha_1 \exp\left(-\frac{\alpha_2}{2}\left|\left|\mathbf{x}_i - \mathbf{x}_j\right|\right|^2\right) + \alpha_3\delta_{ij}.$$

GPLVM

- Comparison

	Proximity	$\mathbf{X} \rightarrow \mathbf{Y}$	$\mathbf{Y} \rightarrow \mathbf{X}$	Non-linear	Probabilistic	Convex
PCA	I	Y	Y		I	Y
FA		Y	Y		Y	Y
Kernel PCA	Y		Y	Y		Y
MDS	Y			Y		
Sammon mapping	Y			Y		
Neuroscale	Y		Y	Y		
Spectral clustering	Y			Y		Y
Density Networks		Y		Y	Y	
GTM		Y		Y	Y	
GP-LVM	I	Y		Y	Y	

Comparison


PCA VS. GPLVM VS. GPDM
Ritsumeikan University
Emergent System Lab.
Liu HaiLong

GPLVM misc.

- Control of Velocity

$$k_{\dot{\mathbf{Y}}}([\mathbf{x}_i, \mathbf{x}_{i-1}], [\mathbf{x}_j, \mathbf{x}_{j-1}]; \bar{\beta}) =$$
$$\beta_1 \dot{\mathbf{x}}_i^{\mathrm{T}} \dot{\mathbf{x}}_j \exp\left(-\frac{\beta_2}{2} \|\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j\|^2 - \frac{\beta_3}{2} \|\mathbf{x}_i - \mathbf{x}_j\|^2 \right) + \beta_4 \delta_{ij},$$

- Model Learning

$$\ln p(\mathbf{X}, \bar{\alpha}, \bar{\beta}, \mathbf{W}, \mathbf{W}_{\dot{\mathbf{Y}}} | \mathbf{Y}, \dot{\mathbf{Y}}) \propto$$
$$\mathcal{L}_{\mathbf{Y}} + \mathcal{L}_{\dot{\mathbf{Y}}} + \Phi_D(\mathbf{X}) + \Phi_C(\mathbf{X}) + \ln p(\bar{\alpha}) + \ln p(\bar{\beta}).$$

- Pose Synthesis

$$g_{\mathbf{Y}}(\mathbf{x}) = \mathbf{W}\mathbf{Y}^{\mathrm{T}} \mathbf{K}_{\mathbf{Y}}^{-1} \mathbf{k}(\mathbf{x}) + \mathbf{b},$$
$$g_{\mathbf{Y}}^{\sigma}(\mathbf{x}) = \mathbf{W}^2 \left(k_{\mathrm{rbf}}(\mathbf{x}, \mathbf{x}) - \mathbf{k}(\mathbf{x})^{\mathrm{T}} \mathbf{K}_{\mathbf{Y}}^{-1} \mathbf{k}(\mathbf{x}) \right),$$

Conclusion

Dimensionality Reduction

- An Open Problem
  - General? Specific?
- The world might be simple
  - Or the principle component might be simple
- Looking for a compact representation
  - Human?
  - Computer?

Thanks